# IMAGE SEGMENTATION THROUGH FUZZY FEATURE SPACE ANALYSIS

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ABSTRACT: We propose an image segmentation algorithm based on local measures and fuzzy feature space analysis. We address the problem of assigning individual pixels to multiple classes based on computed pixel properties. This new segmentation algorithm extends an object/background segmentation approach introduced in 1992. The idea of analyzing trajectories in fuzzy feature space leads to the partitioning of the image. The promising results are compared to a fuzzy entropy approach. We conclude the paper with a discussion of the potential of the approach and directions for possible future extensions.

## 1 INTRODUCTION

Due to the variability of applications and the desired information to be extracted from an image, there is no optimal solution available to image segmentation (Pal and Pal, 1993). Developing approaches that combine aspects of classical and fuzzy mathematical techniques may contribute to a better understanding of the perceptual tasks to be modelled.

The segmentation algorithm proposed in this paper utilises global and local information as well as crisp and fuzzy concepts to achieve pixel classification of the image into multiple image regions. In an iterative process, membership values and neighbourhood information are gained to provide features characterizing each pixel in an image represented as a fuzzy subset. Trajectories in fuzzy feature space are analyzed in order to find fuzzy subsets that lead to a maximum change in image information. These local maxima are interpreted as class boundaries.

This approach, being an extension of an object/background segmentation algorithm according to (Pal and Ghosh, 1992), demonstrates the analytical strength of the combination of multiple features within a fuzzy framework.

## 2 FUZZY SUBSETS OF THE IMAGE PLANE

In the following we use the idea of images defined as fuzzy subsets. Let $X$ denote the image of size $M \times N$. Then a fuzzy subset of $X$ is a mapping $\mu : X \rightarrow [0,1]$, where the membership value $\mu(x,y)$, $(x,y) \in X$, depends on the original feature vector $p(x,y)$ (usually the gray level of the pixel) (Pal, 1992; Krishnapuram and Keller, 1992). For the transformation into fuzzy membership values, the *S-function* as introduced in (Zadeh, 1975) is used:

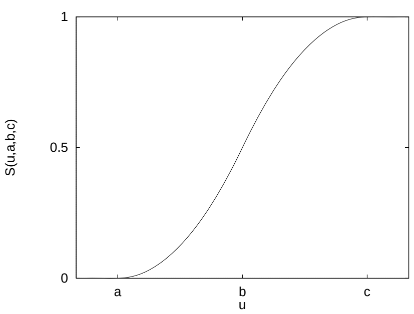



Figure 1: S-function

$$\begin{aligned}\mu(u) &= S(u,a,b,c) \qquad (1)\\ &= \begin{cases} 0 & u \leq a \\ 2\,[(u-a)/(c-a)]^2 & a \leq u \leq b \\ 1 - 2\,[(u-c)/(c-a)]^2 & b \leq u \leq c \\ 1 & u \geq c \end{cases}\end{aligned}$$

with $u = p(x,y)$ being the gray level of a pixel, $2 \cdot \Delta b = c - a$ being the fuzzy region width and $b = (a+c)/2$ denoting the cross-over point. Based on the concept of a fuzzy set, several authors developed measures for the fuzziness in $X$, as for example the fuzzy entropy (see e.g. (Li et al., 1994)).

## 3 FUZZY IMAGE SEGMENTATION

In (Pal and Ghosh, 1992) the pixels are classified according to individually calculated local features. Let $\mu(x, y)$ be the membership value denoting the degree of brightness of a pixel relative to a reference gray level as given from the S-function. It is assumed that the brightness of each pixel is affected by its neighbours. Then the *adjacency* of a pixel (for eight neighbours) is defined as:

$$a(x,y) = \frac{\mu(x,y)}{8}\left(\left(\sum_{j=-1}^{+1}\sum_{i=-1}^{+1}\mu(x-i,y-j)\right)-\mu(x,y)\right) \tag{2}$$

In order to achieve an object/background segmentation, Pal and Ghosh choose $\mu^{2}(x, y)$ and $a(x, y)$ as features (of a pixel) in a 2-D fuzzy feature space:

$$p(x,y) = [\mu^2(x,y), a(x,y)]^T \tag{3}$$

Considering the seed vectors $s(1) = [0, 0]^T$ and $s(2) = [1, 1]^T$ as extreme points in that space, representing object and background (per def. class $C_1$ and class $C_2$) respectively, and choosing them as initial points for an iterative convergence algorithm, results in the *two–class minimum distance classsifier*:

1. Compute feature $\mu^2(x, y)$ for each pixel using the S-function, Eq. (1).
2. Compute feature $a(x, y)$ for each pixel according to Eq. (2).
3. Compute $d(i) = ||p(x, y) - s(i)||$ ; $i = 1, 2$ for each pixel.
4. Assign $p(x, y)$ as follows:

   if $d(1) < d(2)$ then $p(x, y) \in C_1$; (A1)
   if $d(1) > d(2)$ then $p(x, y) \in C_2$;
   if $d(1) = d(2)$ then decide arbitrarily.
5. Replace the seed vectors by the class means $[\overline{\mu}_1^2, \overline{a}_1]^T, [\overline{\mu}_2^2, \overline{a}_2]^T$.
6. Iterate steps 3–6 until the difference between the seed points of two successive iterations is smaller than a limit $\varepsilon$.

(Note that the class means are global features, already.) The question of an optimal segmentation of the two regions is not discussed in (Pal and Ghosh, 1992). To address this problem, we extend algorithm (A1). After the execution of step six, a pair of representatives for both object and background exists. Iterating the S-function for different cross-over points $b$ leads to several such pairs – one for every $b$. These successively generated feature vectors can be interpreted as a trajectory in the 2-D fuzzy feature space evolving over gray scales $b$. A cross-over point, generating a pair of representatives with maximum distance to each other can be considered "optimal" (because regions differ maximally). Therefore we define the following *separation* measure:

$$\sigma(\overline{\mu}_1^2, \overline{a}_1, \overline{\mu}_2^2, \overline{a}_2) = 2 \cdot \left|\left(|0.5 - \overline{\mu}_1^2|\right) \cdot \overline{a}_1 - \left(|0.5 - \overline{\mu}_2^2|\right) \cdot \overline{a}_2\right| \tag{4}$$

A cross-over point $b$ with maximum $\sigma$ represents a binary (i.e object/background) segmentation, which is optimal regarding the selected features. Especially, within a class, we demand homogenity ($\overline{a}_i$) and crispness with regard to the membership value ($|0.5 - \overline{\mu}_i^2|$). The difference desired between classes motivates (4).

Having introduced an optimality criterion into the object/background segmentation algorithm, we proceed with the extension of the algorithm to multiple classes. The assignment of the actual pixel into class $C_1$ or $C_2$ (object/background) is done in step four depending on its feature vector.

Looking at the classes differently, we consider $C_1$ as the "segment–class" and $C_2$ as the class "not finally classified yet". The idea is to further propagate the uncertainty information provided by the use of the S-function and to view the intensity based feature values as interdependent over adjacent scales or transformations via the S-function. The assignment "pixel-to-class" is carried out like in the original algorithm (A1) within the $\varepsilon$-iteration, but those pixels assigned to $C_2$ are reclassified at the next cross-over point $b$. A pixel that has been assigned to $C_1$ in a previous step will never be assigned to class $C_2$ in the following steps (due to the monotony of the generated S-function transformations); i.e. each cross-over point builds a potential class characterized through the corresponding feature vector in $C_1$. Furthermore, class $C_2$ is vanishing as the successive transformation proceeds, leading to a complete classification of the image.

As in (Li et al., 1994) we need a criterion to define the number of classes, their order according to their quality as well as to select class boundaries. Again, we explore the potential of the feature vectors within the fuzzy feature space. At this point, each pixel is assigned to a potential class already, but these have no superior structure or order. Now, we intend to gain different segments, based on these assignments. Looking at the feature vectors $[\overline{\mu}_1^2, \overline{a}_1]^T$ of class $C_1$ we define:

$$\delta([\overline{\mu}_1^2, \overline{a}_1]_i^T, [\overline{\mu}_1^2, \overline{a}_1]_{i+1}^T) = \frac{k_{i+1}}{MN} \left( 1 - \frac{< [\overline{\mu}_1^2, \overline{a}_1]_i^T, [\overline{\mu}_1^2, \overline{a}_1]_{i+1}^T >}{2} \right) \tag{5}$$

where $<,>$ denotes the scalar product and $k_{i+1}$ the number of pixels assigned to class $C_{i+1}$ (relative to the image size $MN$). $\delta$ denotes the change of the feature vector under successive transformation. The cross-over points corresponding to the local maxima of (5) are regarded as *class boundaries*, because they imply a major change in image (region) information represented through feature vectors and region size. Hence, we propose the following algorithm for *multiple class image segmentation*:

1.-6. Execute steps 1-6 of algorithm (A1).

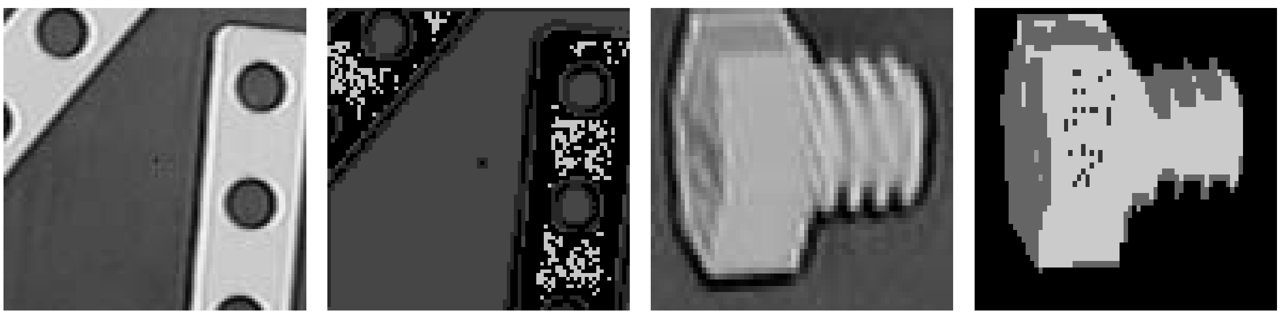
7. Iterate steps 1-7 over all $p(x, y) \in C_2$ and for all $b$ of the S-function.

8. Segment the image according to the potential classification gained from the convergence process (A1) with respect to the class boundaries found from analyzing the trajectory in fuzzy feature space via Eq. (5).

(A2)

## 4 EXPERIMENTAL RESULTS

In order to demonstrate the performance of the algorithm, we show three examples of significantly different complexity in image information. The first example, Fig. 2a, is simply structured and contains little information.



(a) (b) (c) (d)

Figure 2: (a) 'Slats', (b) segmentation result (4 classes), (c) 'screw' and (d) segmentation result (5 classes).

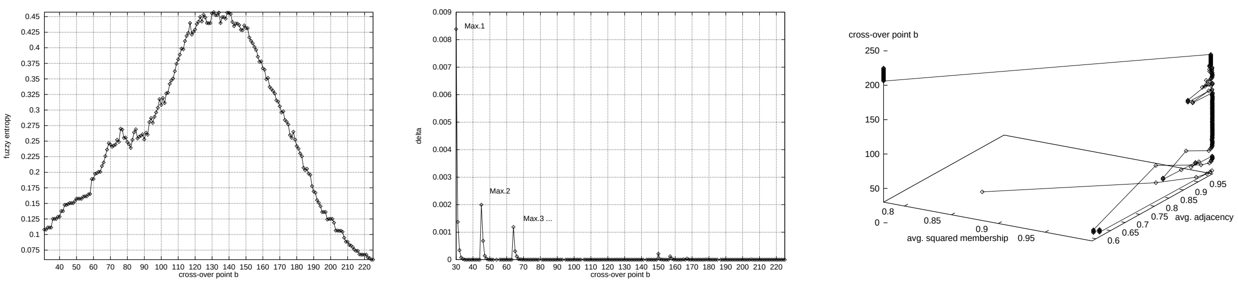
In Fig. 2b the segmentation result using the proposed algorithm (A2) with 4 displayed classes is shown. The extracted classes are obviously meaningful: object, background, shadows and light-reflexions. For the second, more difficult example in Fig. 2c (note the lighting conditions and the resulting gray value transitions), a 5 class-segmentation result is shown in Fig. 2d.

Furthermore, we compare the performance of the derived algorithm to the results of the fuzzy entropy algorithm presented in (Li et al., 1994). For demonstration, we use the Lenna image ($512 \times 480$ resolution, 256 gray levels) in Fig. 3a .

(a) (b) (c) (d)

Figure 3: (a) The unprocessed Lenna image, the segmented image (b) using the fuzzy entropy approach with 3 classes, using the new approach based on trajectory analysis with (c) 3 and (d) 10 classes.

This is a highly complex image containing diffuse structures. Figures 3b and 3c show the results for a 3-class segmentation using the fuzzy entropy approach (Li et al., 1994) and the new multiple class segmentation approach presented in this paper ($2\Delta b = 60$ and $\varepsilon = 0.001$). The curves, which the classifications are based on, are appended in Figures 4a, 4b and 4c.




(a) (b) (c)

Figure 4: (a) The entropy-curve for the segmented image in Fig. 3b, (b) the $\delta$ values according to Eq. (5), and (c) the corresponding trajectory (considering class $C_1$) in fuzzy feature space for the segmented image in Fig. 3c.

As expected, the results are quite different from each other reflecting the use of different features for the classification (although in (Li et al., 1994) the fuzzy set correlation (adjacency measure) is also used), as well as different measures selecting the class boundaries. However, the new approach provides compact "clusters" of identically characterized pixels rather than isolated pixels. Nevertheless, structures in the image are preserved. Consider Fig. 3b and Fig. 3c , for example. Here, the extracted classes consist of simpler structures regarding the image information they represent (Fig. 3c ). In Fig. 3b , on the other hand, classes are less homogenous in structure. From the comparison of Fig. 4a with Fig. 4b one can see that the location of the local maxima, leading to the class boundaries, is of major importance. Figure 4c shows the corresponding trajectory displaying characteristic changes as well as areas which are rather constant in feature values. The changes in feature vectors over the evolution of the cross-over point $b$ imply a major change in pixel-assignment, serving as an indicator for class boundaries.

Fig. 3d shows a 10-class segmentation result of our approach. This example again demonstrates the ability of the new algorithm to perform "sifting" of identically characterized pixels according to the features calculated. Regions tend to be more homogenous and well separated.

## 5 CONCLUSIONS

We presented an image segmentation approach based on fuzzy feature space analysis extending an object/background segmentation approach to multiple classes. Using an iterative convergence algorithm to generate trajectories in fuzzy feature space and analyzing them, the image is segmented into several regions according to the selected features. Due to the use of the S-function, the algorithm is relatively insensitive to noise. The maximum number of classes is determined automatically. The uncertainty of class assignment (as well as the number of classes) can be influenced by changing the fuzzy region width, the adjacency measure or by choosing different (or additional) features to be evaluated. As shown, using a crisp measure to evaluate the trajectory in the fuzzy feature space leads to a fruitful combination of both concepts. The results are evaluated referencing the fuzzy entropy approach. Computation time basically depends on the convergence of the iterative part of the algorithm, particularly the choice of $\varepsilon$.

Possible extensions of the algorithm are the expansion of the fuzzy feature space using additive local measures or including global measures in combination (compactness, index of area coverage, etc.; see e.g. (Pal and Pal, 1993)). Scalability is therefore inherently provided. The convergence of the iterative part may certainly be improved by using techniques other than successive averaging. Furthermore, parallelization of the algorithm can lead to a significant speed-up and synchronously support scalability (Fleury et al., 1996). The algorithm has also been tested on medical images such as magnetic resonance images and on satellite images. Both applications motivate the use of multiple features, because of the complexity of structures desired to be extracted. The ability to combine crisp *and* fuzzy information with respect to a meaningful segmentation of regions of an image will be further explored.

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